Optimum and Near-Optimum Localization of Single and Multiple Targets: Theory and Practice

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Outline

1. Motivations and Basic Problems
2. Optimum Target Localization: Theoretical Cramér-Rao Bound Analysis
4. Nearly Optimum Target Localization: Approximate ML algorithm
5. Application to Measured Data from Xerox-PARC and 29 Palms
6. Conclusions Future Directions

Motivations and Basic Problems

- Acoustic/seismic target(s) location, DOA est., tracking, beamforming, classification, and separation are some basic operations needed in various military applications
  - Near-field: curved wavefront, localization by direct approach
  - Far-field: planar wavefront, DOA estimation, cross-bearing of DOA's to obtain target location
  - Single source vs. multiple sources
  - Wideband signal: frequency-domain processing
- Theoretical "optimum" system performance analysis
  - Cramér-Rao bound analysis: node geometry dependence, signal dependence, other parameters
- Physical prop. of media:array coherency,unknown speed
- Communications vs.computations: Among/inside nodes

Free-Space Single Source Signal Model

Signal model in time-domain:

$$\begin{bmatrix} x_1(n) \\ \vdots \\ x_R(n) \end{bmatrix} = \begin{bmatrix} a_1 s_0(n-t_1) \\ \vdots \\ a_R s_0(n-t_R) \end{bmatrix} + \begin{bmatrix} w_1(n) \\ \vdots \\ w_R(n) \end{bmatrix}.$$

a_p = signal gain due to near-field geometry, unity in far-field

s_0 = wideband source signal

$t_p = ||\mathbf{r}_s - \mathbf{r}_p|| / v$ = time-delay from source to sensor

$\mathbf{r}_s$ = source location, $\mathbf{r}_p$ = location of the pth sensor

v = speed of propagation in length unit per sample

$w_p \sim Normal(0, \sigma^2)$, independent in time and across sensors

Free-Space Single Source Signal Model in Frequency-Domain

- Perform DFT on a block of L time samples:

$$X_p(k) = DFT\{x_p(n)\} = \sum_{n=0}^{L\text{-}1} x_p(n) e^{-j2\pi nk/N}$$

- Signal model in frequency-domain:

$$\mathbf{X}(k) = \mathbf{S}(k) + \eta(k), \quad k = 0, \ldots, N-1$$

$$\mathbf{X}(k) = \left[X_1(k), \ldots, X_R(k)\right]^T, \ \mathbf{S}(k) = S_0(k)\mathbf{d}(k)$$

$$S_0(k) = DFT\{s_0(n)\} \ = \ \text{source signal spectrum}$$

$$\mathbf{d}(k) = \left[a_1 e^{-j2\pi k t_1/N}, \ldots, a_R e^{-j2\pi k t_R/N}\right]^T \ = \ \text{steering vector}$$

$$\eta(k) \ \sim ComplexNormal(0, L\sigma^2 \mathbf{I}_R)$$

- Space-temporal frequency vector: stacking up for all bins

$$\mathbf{X} = \mathbf{G} + \xi$$

$$\mathbf{G} = \left[\mathbf{S}(0)^T, \ldots, \mathbf{S}(N-1)^T\right]^T, \ \xi = \left[\eta(0)^T, \ldots, \eta(N-1)^T\right]^T$$

Cramér-Rao Bound (CRB) Derivation

Fisher Information Matrix (complex form):

$$\mathbf{F} = 2\,\mathrm{Re}\left[\mathbf{H}^H \mathbf{R}_\xi^{-1} \mathbf{H}\right] = \frac{2}{L\sigma^2}\mathrm{Re}\left[\mathbf{H}^H \mathbf{H}\right]$$

Case I: assume known source signal and speed of propagation

the unknown parameter $\Theta = \mathbf{r}_s \quad \Rightarrow \mathbf{H} = \dfrac{\partial \mathbf{G}}{\partial \mathbf{r}_s^T}$

Case II: assume known source signal but unknown speed of propagation

the unknown parameter $\Theta = \left[\mathbf{r}_s^T, v\right]^T \quad \Rightarrow \mathbf{H} = \left[\dfrac{\partial \mathbf{G}}{\partial \mathbf{r}_s^T}, \dfrac{\partial \mathbf{G}}{\partial v}\right]$

Case III: assume unknown source signal but known speed of propagation

the unknown parameter $\Theta = \left[\mathbf{r}_s^T, |\mathbf{S}_0|^T, \boldsymbol{\Phi}_0^T\right]^T \quad \Rightarrow \mathbf{H} = \left[\dfrac{\partial \mathbf{G}}{\partial \mathbf{r}_s^T}, \dfrac{\partial \mathbf{G}}{\partial |\mathbf{S}_0|^T}, \dfrac{\partial \mathbf{G}}{\partial \boldsymbol{\Phi}_0^T}\right]$

source spectrum vector $\mathbf{S}_0 = \left[S_0(0), \ldots, S_0(N-1)\right]^T$

magnitude part: $|\mathbf{S}_0|$, phase part: $\boldsymbol{\Phi}_0$

Cramér-Rao Bound for Source Localization

Case I: $\sigma^2_{\mathbf{r}_s} \geq \mathrm{trace}\left[\mathbf{F}^{-1}_{\mathbf{r}_s}\right]$, $\quad \mathbf{F}_{\mathbf{r}_s} = \zeta \mathbf{A}$

the **array matrix** $\mathbf{A} = \sum_{p=1}^{R} a_p^2 \mathbf{u}_p \mathbf{u}_p^T$

source directional unit vector $\mathbf{u}_p = (\mathbf{r}_s - \mathbf{r}_p)/\|\mathbf{r}_s - \mathbf{r}_p\|$

the scale factor $\zeta = \dfrac{2}{L\sigma^2 v^2} \sum_{k=0}^{N-1} \left(2\pi k |S_0(k)| / N\right)^2$

Case II: $\sigma^2_{\mathbf{r}_s} \geq \mathrm{trace}\left[\mathbf{F}^{-1}_{\mathbf{r}_{s,v}}\right]_{11:DD}$, $\quad \left[\mathbf{F}^{-1}_{\mathbf{r}_{s,v}}\right]_{11:DD} = \dfrac{1}{\zeta}(\mathbf{A} - \mathbf{Z}_v)^{-1}$,

the **penalty matrix** $\mathbf{Z}_v = (1/\mathbf{t}^T \mathbf{A}_a \mathbf{t}) \mathbf{U} \mathbf{A}_a \mathbf{t} \mathbf{t}^T \mathbf{A}_a \mathbf{U}^T$

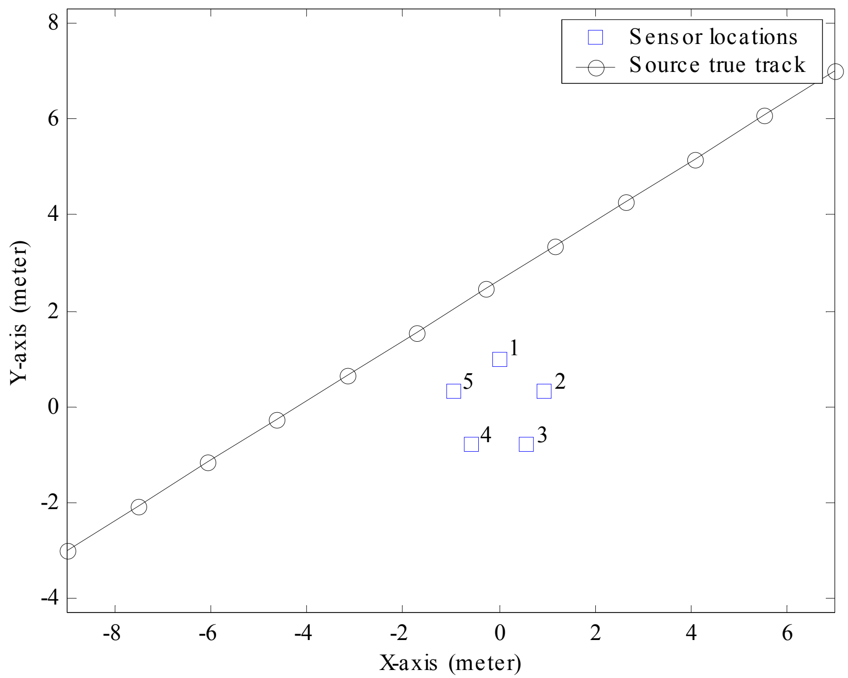
Case III: $\sigma^2_{\mathbf{r}_s} \geq \mathrm{trace}\left[\mathbf{F}^{-1}_{\mathbf{r}_s, \mathbf{S}_0}\right]_{11:DD}$, $\quad \left[\mathbf{F}^{-1}_{\mathbf{r}_s, \mathbf{S}_0}\right]_{11:DD} = \dfrac{1}{\zeta}(\mathbf{A} - \mathbf{Z}_{\mathbf{S}_0})^{-1}$

the **penalty matrix** $\mathbf{Z}_{\mathbf{S}_0} = \dfrac{1}{\sum_{p=1}^{R} a_p^2} \left(\sum_{p=1}^{R} a_p^2 \mathbf{u}_p\right)\left(\sum_{p=1}^{R} a_p^2 \mathbf{u}_p\right)^T$

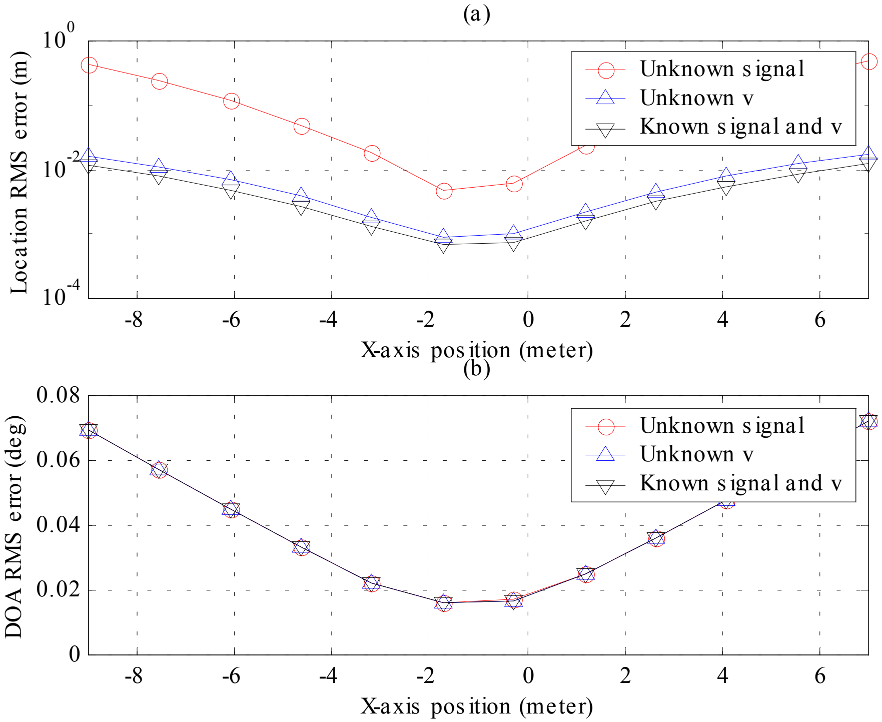
Traveling Target Scenario: Cramér-Rao Bound Numerical Example

- Tracked vehicle signal, circular array of 5 sensors, space loss inversely proportional to square of distance, 12 frames of 200 samples each at $f_s = 1$KHz
- Unknown signal much more significant in range estimation, but not significant in DOA estimation

Traveling target scenario



(a) target location, (b) target DOA



Target Localization Methods

- Two step closed-form method: Least-Squares (LS)
  - Time-delay estimation and then target location estimation
  - Suboptimal, relatively less costly in computation
  - Time-delays are difficult to obtain for multiple targets
- New parametric method: Approximated ML (AML)
  - Directly optimize location estimation
  - Work with multiple sources: alternating projection method
  - Frequency-domain processing
    - Frequency domain signal model is only approximately true due to the artifacts of the DFT, e.g., circular time shift
    - Better frequency domain signal model as time-domain data sample L increases
    - In practice, L is limited by the moving target

Single Target vs. Multiple Targets

- Single target:

  $\mathbf{r}_s = [x_s, y_s]^T$ = target location vector

  AML solution: $\hat{\mathbf{r}}_s = \arg\max_{\mathbf{r}_s} \Upsilon(\mathbf{r}_s), \quad \Upsilon(\mathbf{r}_s) = \sum_{k=1}^{N/2} |\mathbf{d}(k,\mathbf{r}_s)^H \mathbf{X}(k)|^2$

  - Grid-point search
  - Refinement: interpolation, iterative gradient or direct search

- Multiple targets:

  $\tilde{\mathbf{r}}_s = \left[x_{s_1}, y_{s_1}, \ldots, x_{s_M}, y_{s_M}\right]^T$ = target location vector (M sources)

  AM L solution: $\hat{\tilde{\mathbf{r}}}_s = \arg\max_{\tilde{\mathbf{r}}_s} J(\tilde{\mathbf{r}}_s), \; J(\tilde{\mathbf{r}}_s) = \sum_{k=1}^{N/2} \left\| \mathbf{P}(k,\tilde{\mathbf{r}}_s)\mathbf{X}(k) \right\|^2$

  - Alternating projection: sequence of single target parameter search

Alternating Projection (AP) for Multi-Target Case

- Multi-parameter space issues
  - cost, convergence, initial position estimate
- $M = 2$: Step 1: $\mathbf{r}_{s_1}^{(0)} = \arg\max_{\mathbf{r}_{s_1}} J(\mathbf{r}_{s_1})$

Step 2: $\mathbf{r}_{s_2}^{(0)} = \arg\max_{\mathbf{r}_{s_2}} J\left(\left[\mathbf{r}_{s_1}^{(0)^T}, \mathbf{r}_{s_2}^T\right]^T\right)$

For $i = 1, \ldots$

Step 3: $\mathbf{r}_{s_1}^{(i)} = \arg\max_{\mathbf{r}_{s_1}} J\left(\left[\mathbf{r}_{s_1}^T, \mathbf{r}_{s_2}^{(i-1)^T}\right]^T\right)$

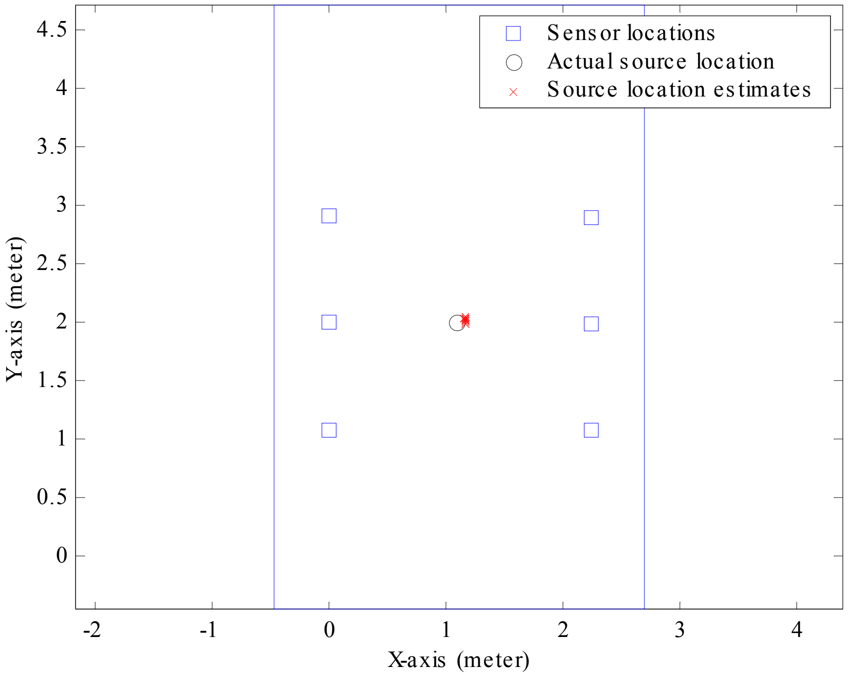
Step 4: $\mathbf{r}_{s_2}^{(i)} = \arg\max_{\mathbf{r}_{s_2}} J\left(\left[\mathbf{r}_{s_1}^{(i)^T}, \mathbf{r}_{s_2}^T\right]^T\right)$

Repeat steps 3 and 4 until convergence

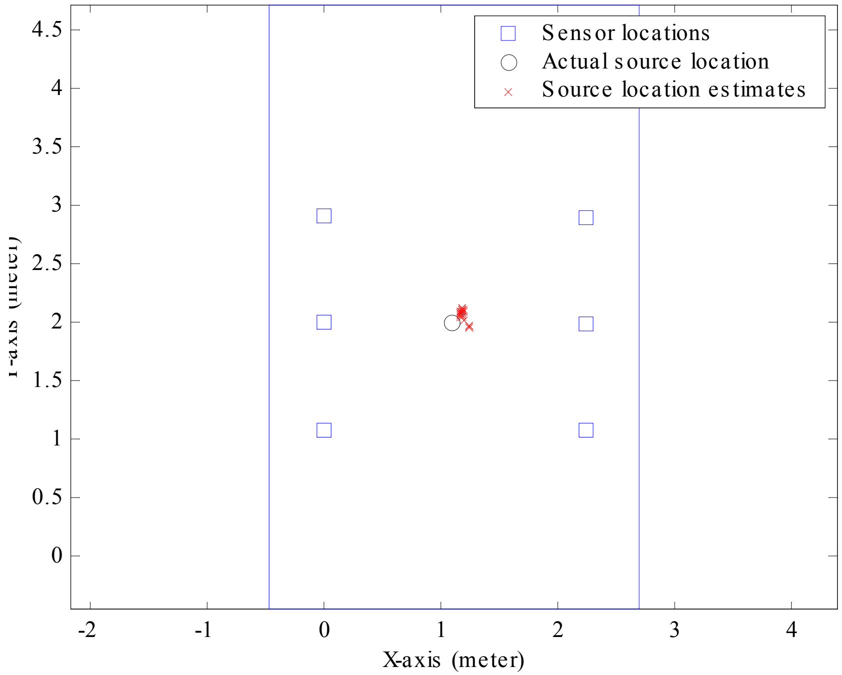
Indoor Convex Hull Experimental Results

- Semi-anechoic room, SNR = 12dB
- Direct localization of an omni-directional loud speaker playing the LAV (light wheeled vehicle) sound
- AML RMS error of 73 cm, LS RMS error of 127cm

AML



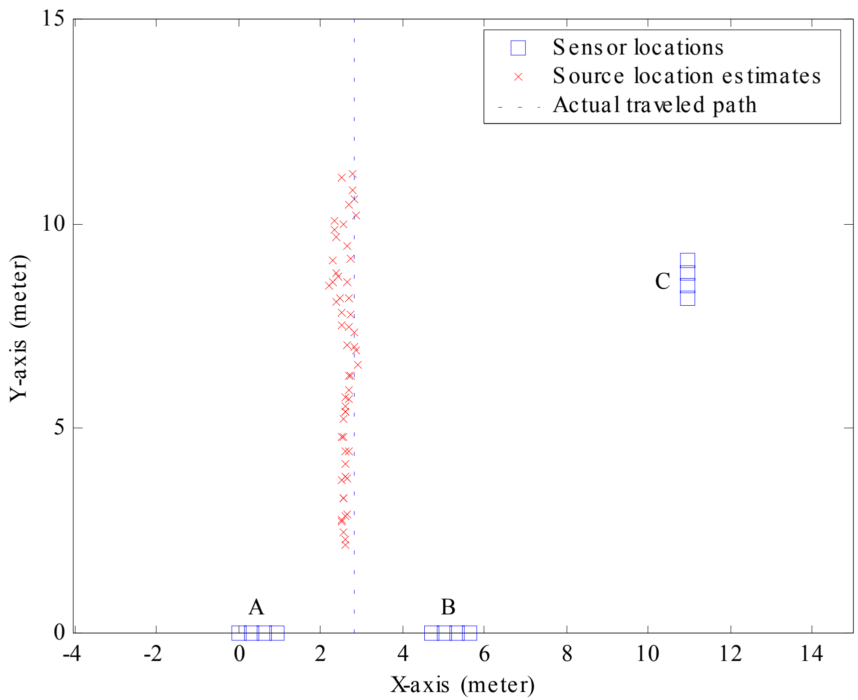
LS



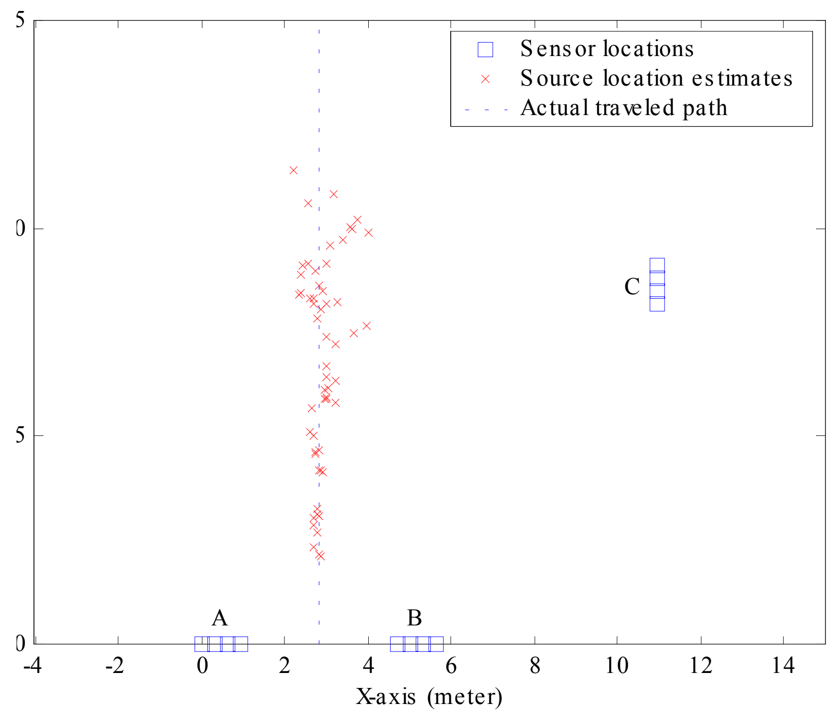
Outdoor Moving Target Experimental Results

- Omni-directional loud speaker playing the LAV sound while moving from north to south
- Far-field situation: cross-bearing of DOA's from three subarrays

AML



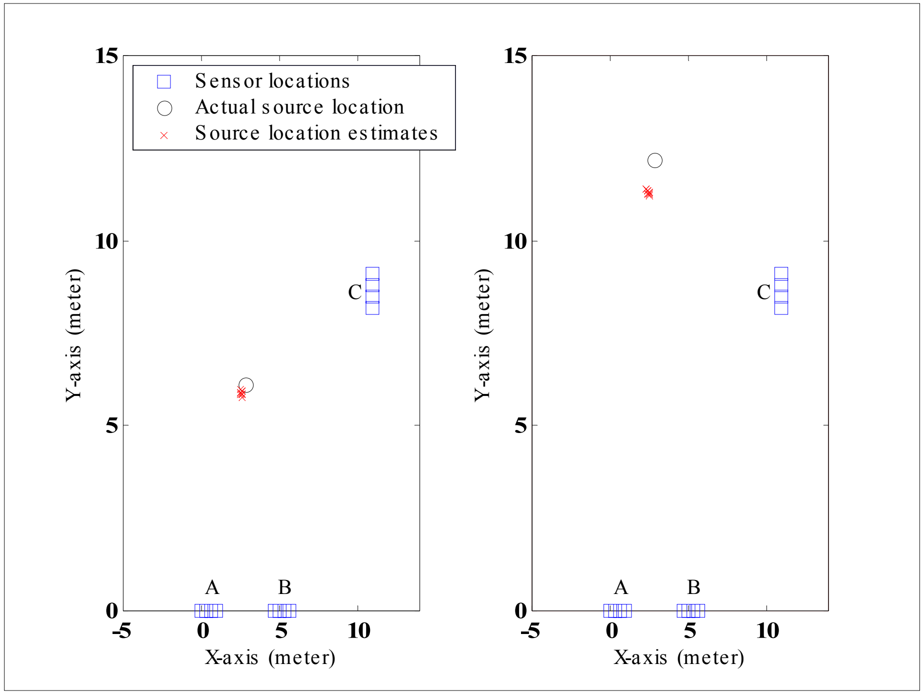
LS



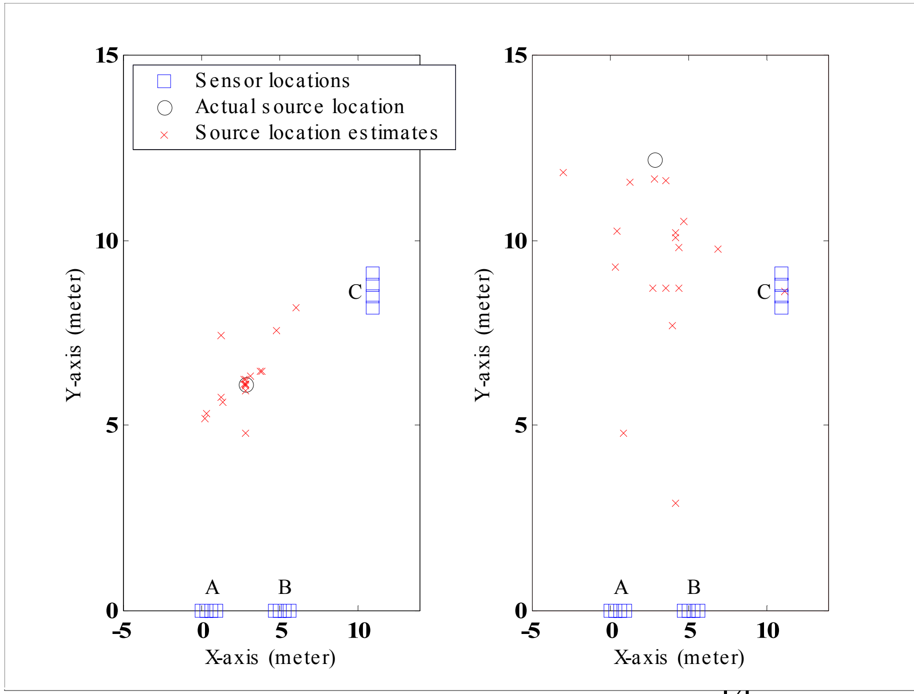
Outdoor Single Source Experimental Results

- Omni-directional loud speaker playing white noise sound
- Cross-bearing of DOA’s from three subarrays
- AML RMS error: 32cm (left) and 97cm (right)
- LS RMS error: 152cm (left) and 472cm (right)

AML

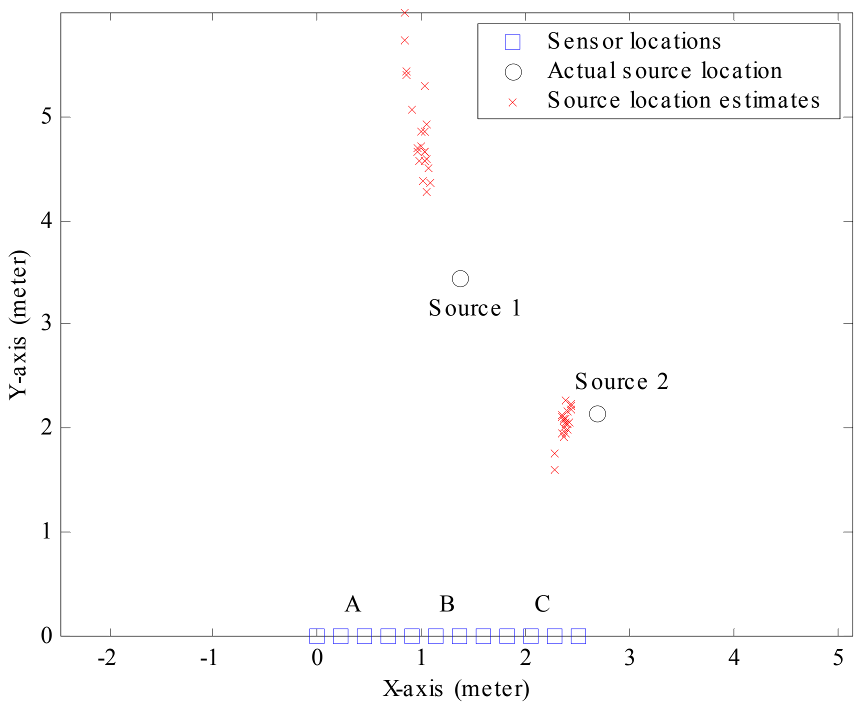


LS



Indoor Two Sources Experimental Result

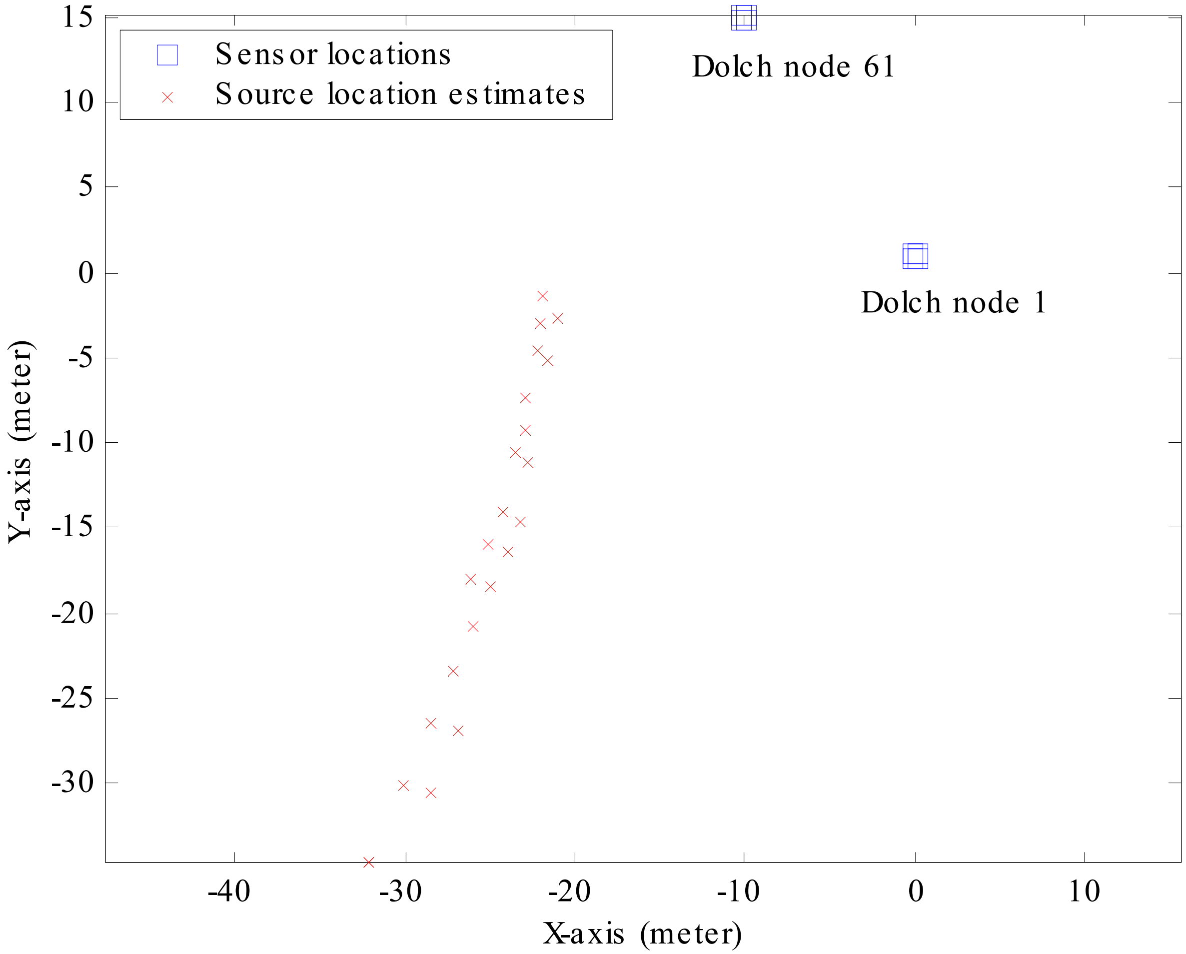
- Semi-anechoic room. One speaker plays LAV and another speaker plays Dragon Wagon (light wheeled vehicle)
- Cross-bearing of DOA's from three subarrays
- RMS error of 154cm (upper) and 35cm (lower)



- AML with alternating projection
- LS method cannot estimate the DOA of multiple sources

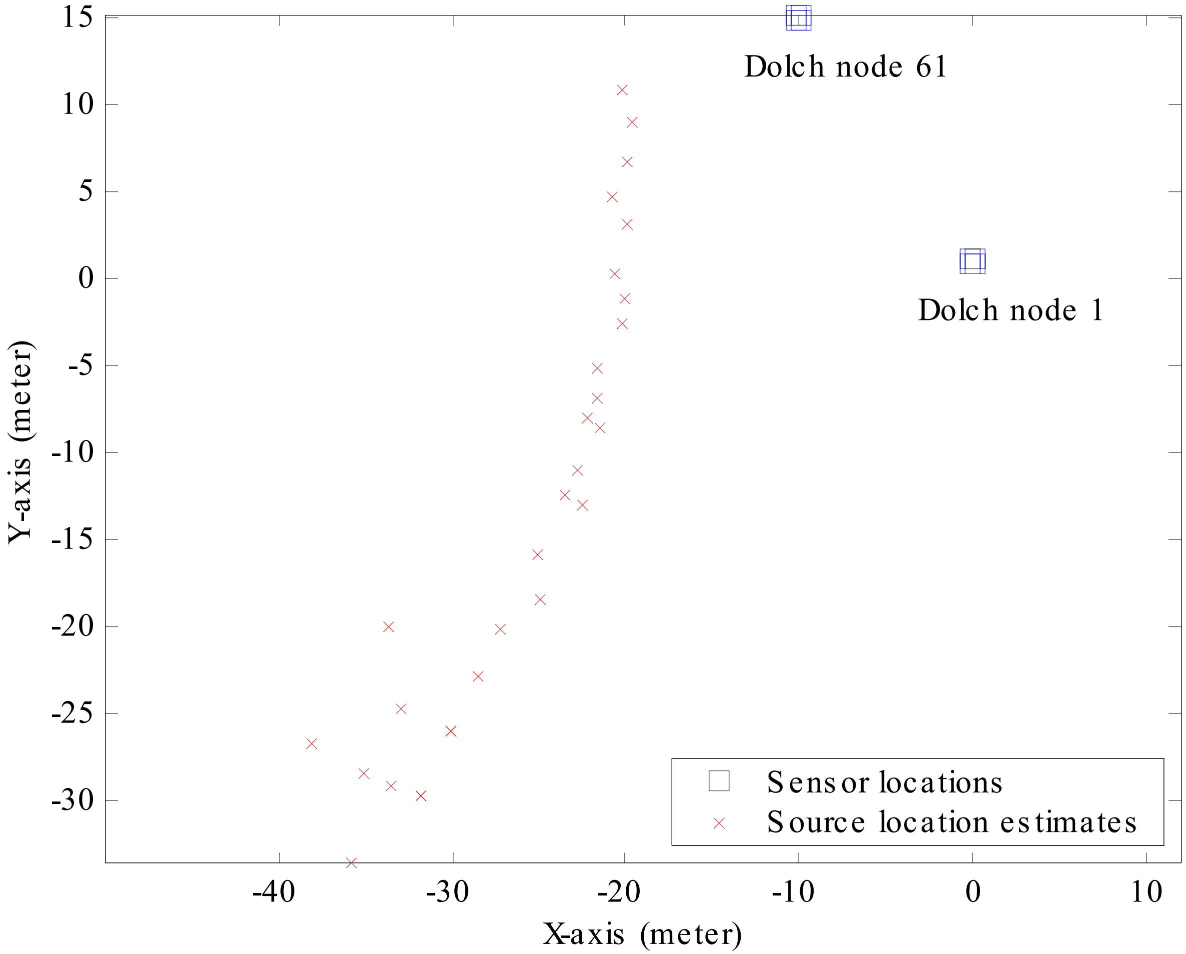
29 Palms Field Measurement Results (1)

- Single Armored Amphibious Vehicle (AAV) traveling at 15mph
- Far-field situation: cross-bearing of DOA's from two subarrays



29 Palms Field Measurement Results (2)

- Single tank traveling at 15mph
- Far-field situation: cross-bearing of DOA's from two subarrays



Conclusions

- CRB analysis
  - Provides mathematical model of opt. array performance
  - Provides theoretical evaluation of sensor placement
- AML target localization
  - Efficient with respect to the CRB
  - Maximizes power in beam-steered beamformer
  - Efficient multi-target algorithm by alternating projection method
- Effective in experimental and field measurement data
  - Direct localization via cross bearing
  - Tracking of single target and two targets

Future Directions

- Physical acoustic/seismic propagation channels are complex
- Acoustic/seismic signal fields are mildly/strongly inhomogeous/non-isotropic among sensor nodes
- Most military scenarios have multiple targets
- We propose to study/find optimum/near optimum and robust localization/beamforming algorithms for multiple targets under the above constraints
- We will address the important autonomous cluster formation of nodes and the minimal density of nodes/unit area problems