

The DARPA DSO mathematics program has a simple strategy to develop the technology necessary to equip the future war fighter. We invest in the development of new mathematical tools in conjunction with an interesting Department of Defense application, forming new communities of mathematicians and subject matter experts to execute our programs. This enables us to rapidly export emerging mathematics to new communities while simultaneously challenging the mathematics community with important defense problems.

One example of this strategy has been in computational electromagnetics where our goal was to compute the radar cross section of an aircraft. At the radar frequencies of interest it would take roughly 10 to the 20 Operations to compute the radar cross section. With today's fastest US supercomputer, we would require at least one year of dedicated computation. Assuming that Moore's Law continues to apply, we will need to wait 15 years before we could perform the computation in a day or less. As a result of challenging the mathematics community with this problem algorithmic solutions were developed that are making these computations commonplace and impacting the design of aircraft. Increases in our computational capability due to Moore's law have made significant contributions to our ability to compute. But the development of fast algorithms has contributed just as much. The development of the Fast Fourier Transform lead to the modern digital world. The Fast Multipole Method has lead to new capabilities in electromagnetic design. In the future we desire fast

methods for elliptic partial differential equations because they will help us design micro fluidic devices, new materials, and more efficient propulsion systems.

DSO's applied and computational mathematics programs is organized along three themes.

The first theme focuses on the development of mathematics for new mission-specific sensor systems. Mathematics will enable us to optimize the , scheduling, collection and interpretation of signals received from complex environments as well as adjusting to dynamic changes in the environment.

The second theme is to enable a capability to predict the properties of large systems and materials. New tools will be needed to allow us to estimate the level of uncertainty of our predictions and assess the limits of our designs

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Our final theme is the reduction of dimensionality or how can we minimize the number of measurements we need to perform in order to collect the most relevant information in a noisy environment and understand the physical world?

Relative to our first theme, mathematics for sensor systems, one ongoing project is the Integrated Sensing and Processing program. A multitude of systems exist for target detection and tracking and they are composed of sensors developed independently of the analysis algorithms and the mission. The Integrated sensing and processing program changes the paradigm, by developing tools that

allow the sensor and the algorithms to maximize mission performance. We are integrating algorithms and sensors so that they will constantly respond to environmental changes and adapt their performance. Our performers have developed a technique that leads to an optimal schedule with look-ahead for multifunctional radars. The algorithm gives the radar the capability to simultaneously track targets, search for new targets and monitor the surrounding environment on a flexible schedule that changes as the mission changes.

Another new tool, signal reversal is being developed. This tool will allow us to exploit multiple scattering, that is, multiple reflections, of RF or acoustic signal energy, to obtain information about the medium and localize sources, to form images, and to communicate overtly or covertly. Signal reversal works a lot like ray tracing, but in reverse. Dealing with multiple scattering is important, because classical image formation methods do not account for highly complex environments, such as the jungle, where acoustic energy is so highly scattered that we don't know the direction from which a sound originates. Development of mathematical signal reversal techniques will allow our war fighters to own the jungle in the same way that night vision goggles gave them ownership of the night.

In the future we see a great need for mathematical techniques and hardware that enable us to generate designer wave forms in support of adaptive signal processing. One of our goals will be developing strategies for choosing a waveform that takes advantage of the propagation channel and simultaneously maximizes information about targets. Imagine that we had some

knowledge of the channel and the target. We can envision sending out a waveform, analyzing the information in the return and then changing the waveform to obtain even more information. Our goal is to create a capability analogous to playing the game of twenty questions with radar and acoustic waveforms and we will be inviting your ideas to give life to our vision.

For the purposes of computation, algorithms are represented in software. We make a choice in developing software – it's either optimized for an architecture or it is portable and runs on many platforms. Optimized software is created by experts hand tuning algorithms for specific processors. This is expensive, time consuming, and leaves us with point solutions. In the future we want software to be simultaneously portable and optimal. Many of the algorithms we use for signal and image processing are constructed from primitive kernels. These primitive kernels can be implemented in many ways – but the space of implementations can be described algebraically. “Algorithmic experiments” suggest that we can implement portable algorithms that are also optimized for the processor architecture. Imagine the impact on our weapon systems of being able to squeeze 100 times more signal and image processing into the constrained environments of an embedded processor with out hand tuning.

Let me shift gears to our second major theme: Predicting the performance of materials and systems. Although the instruments for material design have significantly changed since the 18<sup>th</sup> century, methods such as substitutional chemistry and serendipity have not changed substantially in the intervening years. Our vision is to be able to design materials according to precise

specifications, as opposed to random search through a pallet of available materials. As long as we do not violate any of the constraints imposed by our understanding of physics we believe that can achieve materials with designer properties.

We are beginning to change this paradigm by treating materials design as a mathematical inverse problem. Material properties of interest are expressed mathematically in a form amenable to being “inverted.” While direct inverse methods exist, often we must use fast forward calculations to accomplish an inverse calculation. This poses a significant challenge.

Today we can calculate the properties of simple systems, such as a small nanostructure with at most about 1000 electrons or about 250 atoms. Doubling the number of electrons or atoms, would require sixteen times the compute if we do not change the way in which we do our computations. We need to develop new algorithms that have the property that doubling the number of electrons doubles the compute requirement. A second problem and opportunity is the need for tools that automatically extract parameters across spatial and temporal scales that are used in our simulations that are fast and compatible with optimization techniques.

Another system property that we desire to predict is the RCS, or the way that electromagnetic energy reflects from complex objects, such as a naval surface combatant. For a ship these calculations are beyond our abilities. The calculation is hard because of the size of the naval ship, the materials used in the ship, the environment in which it sails, and the strong interactions between

objects on the ship, and strong interactions between the ship and the rough dynamic sea surface. We want new advanced computational electromagnetics tools that in the next five years solve these problems. We will need your novel algorithmic solutions to make this possible.

The complexity of our systems continues to grow while our capability to analyze and efficiently test these systems has not. Imagine trying to architect a C4ISR network, a heterogeneous swarm of vehicles, the control systems of a modern aircraft, tank, or ship and guarantee performance. These systems are composed of a large number of sensors, actuators, controllers, computer programs, and subsystems. While it is straightforward to hook up all of these pieces, our mathematical tools do not support hooking up the pieces in a way that guarantees that we are able to manage uncertainty arising from small errors in the design, the sensors, actuators, or subsystems. The challenge is to analyze large, complex, stochastic systems, with dynamics on multiple spatial and temporal scales and tools appropriate for this analysis do not exist. Opportunities exist to develop new mathematical methods that will meet these challenges and in turn give us the capability to analyze and design complex systems.

Our final theme is the reduction of dimensionality and has a variety of applications. These are systems like Automatic target recognition suites, analysis tools for biological and medical data sets, as well as data mining and text mining, or any system in which we perform measurements in order to inform a model. In

high dimensional systems it is important to understand the difference between data – i.e. the result of a measurement and information – our interpretation of the data. A significant challenge is to understand how much data to collect in order to maximize our information. In high dimensional spaces, too much data can obscure relevant information, and insufficient data doesn't help develop information. Our mission is to invent a new mathematics to define the minimum number of measurements needed to collect all the relevant information necessary to describe a system and make predictions.

Biological systems such as proteins and protein networks are extremely challenging. Proteins control biological functions and offer us an untapped potential for new materials, chemistry, and functions. The number of naturally occurring proteins is huge but they comprise only a small fraction of all possible proteins. We are planning a new program to design novel proteins tailored for specific uses such as replacing rubber, providing new catalysts, or even producing completely new types of materials, including novel nanodevices.

The sheer size of the protein design space is a major obstacle in exploring and optimizing a design. New computational techniques that can design experiments with millions of variables must be invented. We will need your ideas to make this happen.

My colleagues and I picture a future where we have the ability to predict a warfighters' health from data collected by sensors attached to their bodies.

Imagine a patch that could be affixed to the forehead of an individual, changing

its color according to his or her predicted health over the next 48 hours.

However, new mathematical models and ideas are required to extract predictions and build models from the large amount of sensor data that we can potentially collect.

Another difficult problem of feature vector identification occurs in the dynamics of interacting multi player environments, in particular, the formation of human teams. We need to find the feature vectors that predict how an individual will perform in a given team. This will enable us to optimize a team's performance and construct high performing teams. We can then choose and appropriately train the right individuals so that the team succeeds.

With the tools that you will be developing in the future we will be able to identify strategies for the design of experiments in coupled linear and non-linear systems in high dimensional spaces. We will then be able to probe and understand the biochemistry within a cell, design robust and secure C4ISRT networks, power distribution and communication grids, or interacting swarms of manned and unmanned assets in the battlefield.

During this talk I've shared with you our current themes; The optimization of hardware and algorithms to achieve maximal system performance; the ability to predict the performance of materials and systems; and dimensionality reduction. I've shared with you opportunities for the development of new mathematical tools including strategies for waveform adaptive signal processing, new methods for

the computation of elliptic partial differential equations, uncertainty management, and dimensionality reduction as well as applications in which these tools might apply. These are merely scaffolds we use to organize our thoughts and in the future we will be issuing solicitations for all of these areas. We also look forward to receiving your ideas to turn these thoughts into real programs as well as your ideas for future mathematics programs that bridge the gap.

I'd like to thank you for your attention and introduce Doctor Brett Giroir.