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Through a Mathematical Looking Glass

Galileo famously said, "The book of Nature is an open book, but it is written in mathematical language." History shows that the advancement of mathematics and, indeed, the creation of new mathematics are essential to understanding this open book. Today I'm going to explain why it is imperative DoD develops and deploys new mathematics, how some of this new mathematics might be found, what it might look like, and what it could accomplish.

From Archimedes' war machines to the Manhattan project, from Caesar's ciphers to the Enigma code breakers, from Napoleon's reliance on mathematicians to the development of radar and stealth, mathematical discoveries and implementations have played crucial roles in military history. Part of the beauty of this history is how mathematics and technology enrich each other. New mathematics developed to solve one problem leads to solutions of others. The mathematical community responds to its scientific environment, producing new tools in response to new stimuli.

Mathematics has been tremendously effective in framing and solving problems arising in the hard sciences and engineering while being much less successful in the soft sciences. Today's challenges in the hard sciences and engineering are becoming "unreasonably complicated." There is no better example than the challenges facing DoD in highly mobile network-centric warfare, where massive complexity of systems and design are complicated by multifaceted nonlinear dynamics. Current mathematical challenges in the hard and soft sciences are actually merging as the increasing complexity of each makes them "softly" understood and generally intractable. From the DoD perspective, failing to confront this increasing complexity would be disastrous.

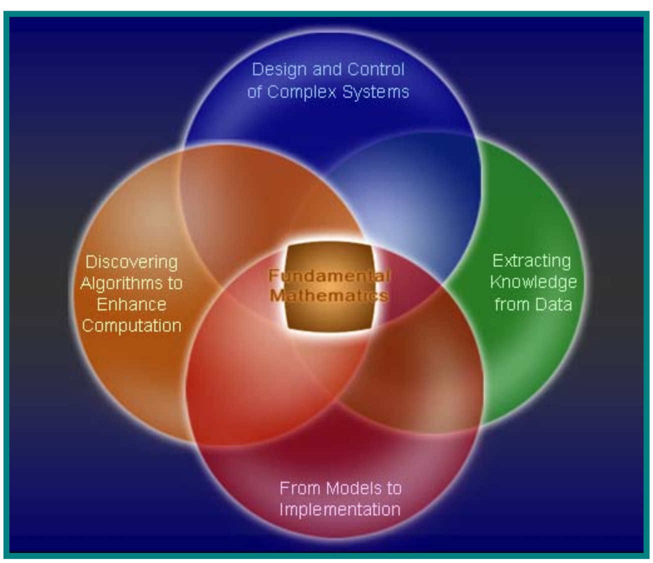
Massive quantities of data being collected today far outstrip DoD's current abilities to understand and act on that information. This problem cuts widely across the military landscape, impacting battlespace awareness, information awareness, and biological issues ranging from Soldier health to bioterrorism. The sheer amount of data is overwhelming. DSO has an innovative approach to understanding massive data sets that will be applied to problems in such diverse areas as network design and analysis, system performance, smart materials, structures and devices, advanced diagnostics, and biological warfare and terrorism defense.

In the 4th century BC, Aristarchus of Samos combined brilliant insight and Euclidean geometry to construct a heliocentric model of the solar system. Centuries later, Ptolemy rejected this model, assumed the earth did not spin on its axis, and used a computationally ingenious method—incompatible with the laws of motion as we know them today—to explain how the sun and stars revolved around the earth. Amassing and interpreting huge amounts of data without understanding the principles underlying that data can be fundamentally misleading. Imagine a global positioning system based on Ptolemy's epi-circles rather than Einstein's theory of relativity. We must not construct the defense systems of the future on technically sophisticated but misguided foundations.

There are many such examples where the laws of nature explain the data, but a particular presentation of the data might hide the laws. One of the great triumphs of Newton's theory of motion was an explanation of the tides. However, it is hard to imagine that anyone could deduce Newton's laws of motion strictly from tidal observations. Further, both Einstein's theory of general relativity and Dirac's prediction of anti-matter follow naturally and relentlessly from key theoretical assumptions. The equations encoding these assumptions forced predictions that, even when the consequences seemed paradoxical and lacked experimental corroboration, later proved to be correct.

Although 2,400 years of thought have produced many theoretical triumphs, Einstein's dream of a unified theory remains elusive. Moreover, recent astronomical research suggests that only 5 percent of the universe consists of matter described by known physical theories. In other words, 95 percent of the matter and energy in the universe remains a mystery.

Biologists are quick to say their subject is far more complex and organizationally intricate than physics. How quickly we forget that, when first proposed, general relativity and quantum mechanics struck the physics community as completely bizarre. In fact, physics now faces some of the same challenges as biology. In biology, it is abundantly clear that elegant principles get lost in real-world examples. The enormous age and time-depth encountered in the life sciences presents problems similar to the physicists' hidden laws: there may be simple truths underlying complicated phenomena, but they may be buried under many layers of structure. It is sobering to realize that biologists today have far more data than pre-Newtonian astronomers, yet the lack of predictive theory remains biology's greatest hurdle. Our Director, Dr. Tether, has established this as a DARPA challenge "to find the fundamental laws of biology." And DSO embraces that challenge.



DSO Mathematical Themes

In DSO, we refuse to believe that biology's descriptive complexity is mathematically intractable. Moreover, there is far more than just biology at stake. If we can develop the tools to understand systems as complex as biology, we can also apply those tools to design complex technological systems, materials, structures, and devices, and even expand our understanding to the dynamics of social networks.

DSO is pursuing an aggressive multipronged mathematics agenda dominated by four major themes. We have made great strides in many directions. However, many DARPA-hard problems remain, and we invite you to be part of these exciting and vital endeavors.

The first major theme is that of Extracting Knowledge from Data. DSO is constructing new geometric representations of geospatial data to support path planning, visibility calculations, 3D modeling, and targeting applications. In a different direction, we are developing new capabilities to analyze massive data sets based on their intrinsic geometries. Our robust geometric methods detect subtle and previously hidden phenomena sampled across a wide variety of applications.

The next major thrust concerns Design and Control of Complicated Systems. In this thrust, DSO pursues the DARPAesque goal of controlling quantum mechanics to develop revolutionary technologies for ultra-secure communications and massive computing applications. In another direction, the end-to-end mathematical methodologies we are developing enhance sensor networks and mobile target identification. They also permit us to exploit the capabilities and agility of modern sensors to rapidly and intelligently transduce the environment.

We are Discovering Algorithms to Enhance Computation. Rather than waiting for hardware to develop, DSO seeks to leap-frog multiple generations of Moore's Law. The goal is to integrate highly efficient mathematical advances to enable design of air vehicles with minimal electromagnetic signatures. Conversely, we are also developing portable algorithms for immediate deployment of critical, high-performance capabilities on new hardware platforms. In addition, we are revolutionizing the stochastic modeling of fundamental physical systems.

Finally, DSO seeks to move From Models to Implementation. We are taking advantage of disorder to deny the enemy the ability to hide by exploiting multiple scattering properties for communications, target detection, and image formation in cluttered space environments. We are manipulating inverse scattering methods to design new materials and leveraging quantum coherence, pulse shaping, and nonlinear mechanisms for remote detection of biological and chemical agents.

Mathematics systematically creates new DoD capabilities. A keen observation of Robert Calderbank explains it well: "Sometimes through heroism you can make something work. However, understanding why it worked, abstracting it, making it a primitive is the key to getting to the next order of magnitude of scale." This is why mathematics, with its unique and powerful way of distilling knowledge, is absolutely essential to DoD.

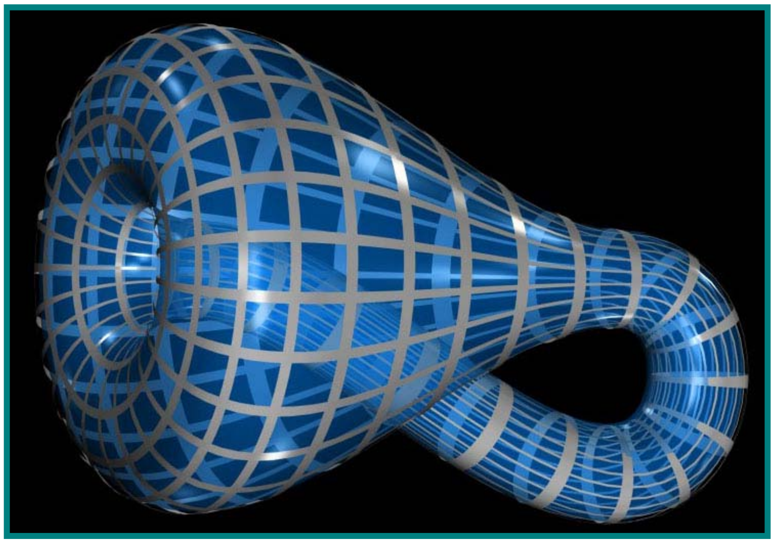
Fundamental mathematics is the glue that holds these themes together. And duality—in which deep connections between seemingly disparate areas of mathematics are uncovered and exploited—is a unification principle for mathematics. Duality enriches computational capabilities dramatically by transforming problems that seem impossible in one context into tractable tasks in others. For example, as in the famous Gauss-Bonnet theorem, an integral calculation capturing the total curvature of a surface can be transformed into a counting problem in topology. Such applications abound in numerical analysis, oscillatory and exponential sums, rapid integration, and discrete optimization.

Sometimes the benefits of duality are delightfully unexpected. DSO-supported mathematicians are investigating interconnections among deep, abstract facts in number theory and harmonic analysis. Subtle geometry related to these interconnections links these investigations to a brave new world of differential equations that, in turn, are uncovering new phenomena in quantum field theory. One of the surprises this research reveals is that this geometry encodes an infinite collection of graphs, each of which simultaneously optimizes connectedness and sparseness. These constructions generalize to higher dimensions yielding other infinite collections of higher-dimensional network-like complexes with the potential to advance secure communications and network design, from the nano level to complicated systems.

Duality and symmetry are two potent reasons why mathematics has traditionally been so successful in the hard sciences. However, they have not yet been successfully employed in the soft sciences. Faced with the exponentially increasing complexity in both the hard and soft sciences and the resulting challenges, DoD must employ more than traditional mathematics. New approaches and new tools are essential. We hope you will join us in developing these instruments and in using them to decipher the heretofore inscrutable secrets of the universe. Some of those new tools are evolving out of DSO's Topological Data Analysis program.

As an astronomer uses a telescope to examine the night sky at different magnifications, this program uses sophisticated methods of algebraic topology to calculate the geometric features of a data set at multiple scales. In this way, one obtains a multiparameter histogram for each data set encoding its statistical structure. Based on this idea, let me make a bold prediction. In traditional mathematics, numbers organized along the real number line are the basic building blocks. In the 21st century, data sets with their associated histograms will replace numbers as the fundamental objects. By replacing numbers with data sets, all of mathematics can be recast into a stochastic framework.

Finally, just as you might rotate an object in space to get a better view of all its features, it will be necessary to rotate or perturb the geometry on a given data set to extract important features. Borrowing from deformation theory in algebraic geometry, individual data sets will be embedded in parameterized families of such sets in such a way that all the associated tangent structures are compatible. Using the tangent structures to determine directions in the parameter space, the individual set will then be transported until its hidden features are best displayed and detected. When all this comes to fruition, the "noisy" or stochastic world of data set mathematics will have sufficient structure to incorporate the symmetries and dualities of classical mathematics.



From its frontal view, the object appears as a torus, or donut-shaped object. Once the object is rotated the full Klein-bottle object appears.

When Dr. Tether asked "what are the fundamental laws of biology?" he concisely framed the great scientific challenge of the 21st century. Biologists have learned the alphabet. They have found the pages in the book of Nature that describe the double-helix and the human genome. So far, no human can read these pages with understanding sufficient to support prediction. Yet we know where the answer lies, for Galileo's insight still guides us: The book of Nature remains written in the mathematics we have yet to discover.